**Lab 3：Fourier Series Representation of Periodic Signals**

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| **Introduction**  **3.9 Frequency Response of a Continuous-Time System**  This exercise demonstrates the effect of the frequency response of a continuous-time system  on periodic signals. You will examine the response of a simple linear system to each of the  harmonics that compose a periodic signal as well as to the periodic signal itself.  **3.10 Computing the Discrete-Time Fourier Series**  Calculate the time spend on calculating the DTFS for a periodic discrete-time signal  **Lab results & Analysis**：    Question(a)    Results    Analysis  Every ak need to sum up all the x[n]\*exp(-jk(2pi/N)n) in a N period, which costs n-1 complex addition and n complex product. Eventually, ak = (1/N)\*Sum, so there is another complex product. So the final complex addition is N-1 and the complex product is N+1;  picture whose x stands for N is above:  The red line is the times that complex product happens, while the other is for the complex addition. We could find that complex product happens N+1 times, while the other happens N-1 times.  Question(b)    Results    Analysis  As the flop function has been removed from matlab, we choose timeit function to calculate the time cost. This is the time cost line of different N.  Question(c)    Results    Analysis  For unknown reasons, I am sorry that I can not handle this.  Question(d)    Results    Analysis  Ny = N.  We set N = 5,10,15,20,25.. The blue line is the count times. The red line is N.^2. And the count times that it needs are about O(N^2).  Question(e)    Results    Analysis  This is the picture of y. And the time it costs is about 0.0155 s for 1000 circles.  Question(f)    Results  Analysis  Question(g)  Results  Analysis  Question(h)  Results  Analysis  Question(i)  Results  Analysis    Question(a)    Results    Analysis  The red line is x = cos(t), and the blue line is y = x\*H.  It can be detected that the amplitude is decreased and the phase change is lead. Because H(s) = 1/(1 + jw), which would decrease the amplitude and its phase shift is -arctan(f\*RC) = -arctan(1),  Question(b)    Results    Analysis  The picture is showed above.  Question(c)    Results    Analysis  x2(t) = s(t) - s(t-T/2) = s(t) - s(t-pi). according to linearity, a(k) should be (sin(k\*pi/2)/(pi\*k))\*(1-exp(-j\*k\*1\*pi)); Then we can get a vector of a(k).  The green line is from the CTFS of x2, and the red line is x2 itself. Their amplitude are not fit very much. And the reason for that is that we only pick 5 harmonic components instead of all harmonic components in a consecutive periodic signal.  Question(d)    Results    Analysis  The green line is the sum from y1 to y5, the red line is the output of the whole S(t). They are the same. So the linear property can be confirmed.  Question(e)    Results    Analysis  The red line comes from the sum of first 5 harmonics to the response of the system. The green line comes from the response of system. They look similar because the energy of CTFS is the accumulate of abs(components.^2), and there are only 10 components’ energy is calculated. So the energy approximately equal to the energy X2 equipped.  The graphs of them would look like the same.  Question(f)    Results    Analysis  For each graph, red line is the analytically determined signal and the green line is the simulated signal. The two signals are the same. And from up to down, they are the graphs of y1,y2,y3,y4,y5 separately.  **Note**: Please indicate meaning of the symbols in all expressions. Please indicate the coordinate and unit in all figures. | |
| **Experience**  **微信图片_20220408135006微信图片_20220408135037**  3.9 is a bit challenging, applying theory to practice, especially the analytical formulation’s construction.  3.10 is more difficult than other questions. It need both a good understanding of Signals and System and mastered Matlab programming. So, I can not complete them all. | |
| **Score** | 80 |

**Code**

**3.9**

(a)

clc;clear;

t = linspace(0,20,1000);

x = cos(t);

y = lsim([1],[1,1],x,t);

plot(t,x,'r');

hold on;

plot(t,y,'b');

xlim([10 20]);

xlabel("10<=t<=20");

ylabel("x=cost and y=H\*x");

legend([plot(t,x,'r'),plot(t,y,'b')],'x=cost','y=H\*x');

grid;

(b)

clc;clear;

t = linspace(0,20,1000);x2 = cos(t);

x2(x2>0) = ones(size(x2(x2>0)));

x2(x2<0) = -ones(size(x2(x2 < 0)));

y2 = lsim([1],[1,1],x2,t);

plot(t,y2);grid;xlabel("10<=t<=20");ylabel("y2 = H\*x2");xlim([10 20]);

(c)

clc;clear;

t = linspace(0,20,1000);x2 = cos(t);

x2(x2>0) = ones(size(x2(x2>0)));

x2(x2<0) = -ones(size(x2(x2 < 0)));

k = -1;

for i = 1:5

aneg\_k(i) = (sin((k\*pi)/2)/(pi\*k))\*(1-exp(-j\*k\*1\*pi));

k = k-2;

end

k = 1;

for i = 1:5

apos\_k(i) = (sin((k\*pi)/2)/(pi\*k))\*(1-exp(-j\*k\*1\*pi));

k = k+2;

end

s1 = apos\_k(1)\*exp(j\*1\*t) + aneg\_k(1)\*exp(-j\*1\*t);

s2 = apos\_k(2)\*exp(j\*3\*t) + aneg\_k(2)\*exp(-j\*3\*t);

s3 = apos\_k(3)\*exp(j\*5\*t) + aneg\_k(3)\*exp(-j\*5\*t);

s4 = apos\_k(4)\*exp(j\*7\*t) + aneg\_k(4)\*exp(-j\*7\*t);

s5 = apos\_k(5)\*exp(j\*9\*t) + aneg\_k(5)\*exp(-j\*9\*t);

s = s1 + s2 + s3 + s4 + s5;

plot(t,x2,'r--');xlabel("0<=t<=20");ylabel("x2 and its CTFS");

hold on

plot(t,s,'g');

legend([plot(t,x2,'r--'),plot(t,s,'g')],'x2 = square wave','s = CTFS of square wave');

grid;

(d)

clc;clear;

t = linspace(0,20,1000);x2 = cos(t);

x2(x2>0) = ones(size(x2(x2>0)));

x2(x2<0) = -ones(size(x2(x2 < 0)));

k = -1;

for i = 1:5

aneg\_k(i) = (sin(k\*pi/2)/(pi\*k))\*(1-exp(-j\*k\*1\*pi));

k = k-2;

end

k = 1;

for i = 1:5

apos\_k(i) = (sin(k\*pi/2)/(pi\*k))\*(1-exp(-j\*k\*1\*pi));

k = k+2;

end

s1 = apos\_k(1)\*exp(j\*1\*t) + aneg\_k(1)\*exp(-j\*1\*t);

s2 = apos\_k(2)\*exp(j\*3\*t) + aneg\_k(2)\*exp(-j\*3\*t);

s3 = apos\_k(3)\*exp(j\*5\*t) + aneg\_k(3)\*exp(-j\*5\*t);

s4 = apos\_k(4)\*exp(j\*7\*t) + aneg\_k(4)\*exp(-j\*7\*t);

s5 = apos\_k(5)\*exp(j\*9\*t) + aneg\_k(5)\*exp(-j\*9\*t);

ssum = s1 + s2 + s3 + s4 + s5;

y1 = lsim([1],[1,1],s1,t);

y2 = lsim([1],[1,1],s2,t);

y3 = lsim([1],[1,1],s3,t);

y4 = lsim([1],[1,1],s4,t);

y5 = lsim([1],[1,1],s5,t);

ysum = lsim([1],[1,1],ssum,t);

y12345 = y1 + y2 + y3 + y4 + y5;

plot(t,ysum,'r\*');xlabel("0 <= t <= 20");ylabel("ySum =y1+y2+y3+y4+y5, yWhole=H\*sSum");

hold on;plot(t,y12345,'g','linewidth',1.5);legend([plot(t,ysum,'r\*'),plot(t,y12345,'g','linewidth',1.5)],'yWhole = H\*sSum','ySum = y1+y2+y3+y4+y5');

(e)

clc;clear;

t = linspace(0,20,1000);x2 = cos(t);

x2(x2>0) = ones(size(x2(x2>0)));

x2(x2<0) = -ones(size(x2(x2 < 0)));

k = -1;

for i = 1:5

aneg\_k(i) = (sin(k\*pi/2)/(pi\*k))\*(1-exp(-j\*k\*1\*pi));

k = k-2;

end

k = 1;

for i = 1:5

apos\_k(i) = (sin(k\*pi/2)/(pi\*k))\*(1-exp(-j\*k\*1\*pi));

k = k+2;

end

s1 = apos\_k(1)\*exp(j\*1\*t) + aneg\_k(1)\*exp(-j\*1\*t);

s2 = apos\_k(2)\*exp(j\*3\*t) + aneg\_k(2)\*exp(-j\*3\*t);

s3 = apos\_k(3)\*exp(j\*5\*t) + aneg\_k(3)\*exp(-j\*5\*t);

s4 = apos\_k(4)\*exp(j\*7\*t) + aneg\_k(4)\*exp(-j\*7\*t);

s5 = apos\_k(5)\*exp(j\*9\*t) + aneg\_k(5)\*exp(-j\*9\*t);

ssum = s1 + s2 + s3 + s4 + s5;

ysum = lsim([1],[1,1],ssum,t);

y2 = lsim([1],[1,1],x2,t);

plot(t,ysum,'r\*');xlabel("0 <= t <= 20");ylabel("ySum comes from first five CTFS, y2=H\*X2");

hold on;plot(t,y2,'g','linewidth',1.5);legend([plot(t,ysum,'r\*'),plot(t,y2,'g','linewidth',1.5)],'ySum =H \* first five CTFS','y2=H\*X2');

(f)

clc;clear;

t = linspace(0,20,1000);x2 = cos(t);

x2(x2>0) = ones(size(x2(x2>0)));

x2(x2<0) = -ones(size(x2(x2 < 0)));

k = -1;

for i = 1:5

aneg\_k(i) = (sin(k\*pi/2)/(pi\*k))\*(1-exp(-j\*k\*1\*pi));

k = k-2;

end

k = 1;

for i = 1:5

apos\_k(i) = (sin(k\*pi/2)/(pi\*k))\*(1-exp(-j\*k\*1\*pi));

k = k+2;

end

s1 = apos\_k(1)\*exp(j\*1\*t) + aneg\_k(1)\*exp(-j\*1\*t);

s2 = apos\_k(2)\*exp(j\*3\*t) + aneg\_k(2)\*exp(-j\*3\*t);

s3 = apos\_k(3)\*exp(j\*5\*t) + aneg\_k(3)\*exp(-j\*5\*t);

s4 = apos\_k(4)\*exp(j\*7\*t) + aneg\_k(4)\*exp(-j\*7\*t);

s5 = apos\_k(5)\*exp(j\*9\*t) + aneg\_k(5)\*exp(-j\*9\*t);

y1 = lsim([1],[1,1],s1,t);

y2 = lsim([1],[1,1],s2,t);

y3 = lsim([1],[1,1],s3,t);

y4 = lsim([1],[1,1],s4,t);

y5 = lsim([1],[1,1],s5,t);

z1 = apos\_k(1)\*(1/(1+j\*1)).\*exp(j\*1\*t) + aneg\_k(1)\*(1/(1-j\*1)).\*exp(-j\*1\*t);

z2 = apos\_k(2)\*(1/(1+j\*3)).\*exp(j\*3\*t) + aneg\_k(2)\*(1/(1-j\*3)).\*exp(-j\*3\*t);

z3 = apos\_k(3)\*(1/(1+j\*5)).\*exp(j\*5\*t)+ aneg\_k(3)\*(1/(1-j\*5)).\*exp(-j\*5\*t);

z4 = apos\_k(4)\*(1/(1+j\*7)).\*exp(j\*7\*t) + aneg\_k(4)\*(1/(1-j\*7)).\*exp(-j\*7\*t);

z5 = apos\_k(5)\*(1/(1+j\*9)).\*exp(j\*9\*t) + aneg\_k(5)\*(1/(1-j\*9)).\*exp(-j\*9\*t);

subplot(511);plot(t,y1,'r\*');xlabel("10<=t<=20");ylabel("y1 and a(+-1)->b(+-1)");hold on; plot(t,z1,'g','linewidth',1.5);legend([plot(t,y1,'r\*'),plot(t,z1,'g','linewidth',1.5)],'y1','a(+-1)->b(+-1)');xlim([10 20]);hold off;

subplot(512);plot(t,y2,'r\*');xlabel("10<=t<=20");ylabel("y2 and a(+-3)->b(+-3)");hold on; plot(t,z2,'g','linewidth',1.5);legend([plot(t,y2,'r\*'),plot(t,z2,'g','linewidth',1.5)],'y2','a(+-3)->b(+-3)');xlim([10 20]);hold off;

subplot(513);plot(t,y3,'r\*');xlabel("10<=t<=20");ylabel("y3 and a(+-5)->b(+-5)");hold on; plot(t,z3,'g','linewidth',1.5);legend([plot(t,y3,'r\*'),plot(t,z3,'g','linewidth',1.5)],'y3','a(+-5)->b(+-5)');xlim([10 20]);hold off;

subplot(514);plot(t,y4,'r\*');xlabel("10<=t<=20");ylabel("y4 and a(+-7)->b(+-7)");hold on; plot(t,z4,'g','linewidth',1.5);legend([plot(t,y4,'r\*'),plot(t,z4,'g','linewidth',1.5)],'y4','a(+-7)->b(+-7)');xlim([10 20]);hold off;

subplot(515);plot(t,y5,'r\*');xlabel("10<=t<=20");ylabel("y5 and a(+-9)->b(+-9)");hold on; plot(t,z5,'g','linewidth',1.5);legend([plot(t,y5,'r\*'),plot(t,z5,'g','linewidth',1.5)],'y5','a(+-9)->b(+-9)');xlim([10 20]);hold off;

**3.10**

(a)

clc;clear;

for m = 1:20

x = [0:m];

n\_init = 0;

for k = 1:length(x)

b = 0;product = 0;add = 0;

for nn = 1:length(x)

b = b + x(nn)\*exp(-1\*j\* (nn + n\_init - 1) \*(2\*pi/length(x)) \* (k - 1));product = product + 1;if(nn ~= 1)add = add + 1;end

end

ak(k) = (1/length(x)) \* b;product = product+1;

end

p(m) = product;

ad(m) = add;

end

stem([2:21],p,'r');xlabel("x = the fundamental period N");ylabel("yRed = complex product, yGreen = complex addition");hold on; stem([2:21],ad,'g');legend([stem([2:21],p,'r'),stem([2:21],ad,'g')],'yRed = complex product','yGreen = complex addition');

(b)

clc;clear;

N = [8,32,64,128,256];for i = 1:length(N)

x = 0.9.^[0:N(i)-1];f = @() dtfs(x,0);

dtfscomps(i) = timeit(f);

end

plot([1:5],dtfscomps);xlabel("N = 8, 32, 64, 128, 256");ylabel("the time cost in different N by using dtfs function"); set(gca,'xticklabel',N); set(gca,'xtick',[1 2 3 4 5]);

function ak = dtfs(x,n\_init)

for k = 1:length(x)

b = 0;

for nn = 1:length(x)

b = b + x(nn)\*exp(-1\*j\* (nn + n\_init - 1) \*(2\*pi/length(x)) \* (k - 1));

end

ak(k) = (1/length(x)) \* b;

end

end

(c)

clc;clear;

N = [8,32,64,128,256];

for i = 1:length(N)

x = 0.9.^(0:N(i)-1);

f = @() dtfs(x,0);

dtfscomps(i) = timeit(f);

end

for i = 1:length(N)

x = 0.9.^(0:N(i)-1);

f = @() (1/N(i)).\*fft(x);

fftcomps(i) = timeit(f);

end

plot(dtfscomps);

set(gca,'xticklabel',N);

set(gca,'xtick',[1 2 3 4 5]);

hold on;

plot(fftcomps);

xlabel('N');

ylabel('the time cost. blue line is dtfs, red line is fft');

(d)

clc;clear;

N = [5,10,15,20,25];

for i = 1:length(N)

op(i) = 0;

x = [1:N(i)];

h = [1:N(i)];

for n = 1:length(x)

y(n) = 0;

for r = 1 :length(x)

if(r ~= 1)op(i) = op(i) + 1; end;if(n-r < 1 | n - r > length(x))continue;end;

y(n) = y(n) + x(r)\*h(abs(n-r));

op(i) = op(i) + 1;

end

end

period(i) = length(y);

end

plot(N,op);

xlabel("x = N");ylabel("the count times");

hold on;

b = N.^2;

plot(N,b);

(e)

clc;clear;

x = 0.9.^[0:39];

h = 0.5.^[0:39];

y = conv([x x],h);

y = y(1:40);

stem(y);xlabel('Ny = 40');ylabel('the period convolution of x and h');

f = @() calcon(x,h);

f40c=timeit(f);

function y = calcon(x,h)

for i=1:1000

y = conv([x x],h);

y = y(1:80);

end

end

(f)

(g)

(h)

(i)